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Chapter 23

SINGLE-PHASE INDUCTION MACHINES: THE BASICS

23.1 INTRODUCTION

Most small power (generally below 2 kW) induction machines have to operate with single-phase a.c. power supplies that are readily available in homes, and remote rural areas. When power electronics converters are used three phase a.c. output is produced and thus three phase induction motors may still be used.

However, for constant speed applications (the most frequent situation), the induction motors are fed directly from the available single-phase a.c. power grids. In this sense, we call them single phase induction motors.

To be self-starting, the induction machine needs a travelling field at zero speed. This in turn implies the presence of **two windings** in the stator, while the rotor has a standard squirrel cage. The first winding is called the **main winding** while the second winding (for start, especially) is called **auxiliary winding**.

Single phase IMs may run only on the main winding once they started on two windings. A typical case of single phase single-winding IM occurs when a three IM ends up with an open phase (Chapter 7). The power factor and efficiency degrade while the peak torque also decreases significantly.

Thus, except for low powers (less than $\frac{1}{4}$ kW in general), the **auxiliary winding** is active also during running conditions to improve performance.

Three types of single-phase induction motors are in use today:

- Split-phase induction motors
- Capacitor induction motors
- Shaded-pole induction motors

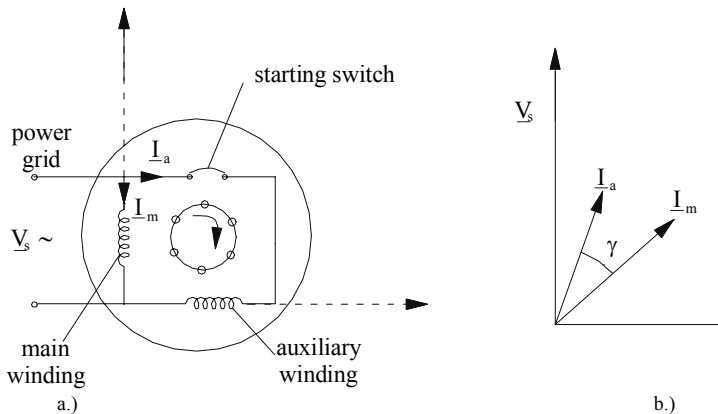


Figure 23.1 The split-phase induction motor

23.2 SPLIT-PHASE INDUCTION MOTORS

The split phase induction motor has a main and an auxiliary stator winding displaced by 90° or up to 110 - 120° degrees (Figure 23.1a).

The auxiliary winding has a higher ratio between resistance and reactance, by designing it at a higher current density, to shift the auxiliary winding current I_a ahead of main winding current I_m (Figure 23.1b).

The two windings-with a 90° space displacement and a $\gamma \approx 20$ - 30° current time phase shift-produce in the airgap a magnetic field with a definite forward travelling component (from m to a). This travelling field induces voltages in the rotor cage whose currents produce a starting torque which rotates the rotor from m to a (clockwise on Figure 23.1).

Once the rotor catches speed, the starting switch is opened to disconnect the auxiliary winding, which is designed for short duty. The starting switch may be centrifugal, magnetic, or static type. The starting torque may be up to 150% rated torque, at moderate starting current, for frequent starts long-running time applications. For infrequent starts and short running time, low efficiency is allowed in exchange for higher starting current with higher rotor resistance.

During running conditions, the split-phase induction motor operates on one winding only and thus it has a rather poor power factor. It is used below 1/3 kW, generally, where the motor costs are of primary concern.

23.3 CAPACITOR INDUCTION MOTORS

Connecting a capacitor in series with the auxiliary winding causes the current in that winding I_a to lead the current in the main winding I_m by up to 90° .

Complete symmetrization of the two windings m.m.f. for given slip may be performed this way.

That is a pure travelling airgap field may be produced either at start ($S = 1$) or at rated load ($S = S_n$) or somewhere in between.

An improvement in starting and running torque density, efficiency and especially in power factor is brought by the capacitor presence. Capacitor motors are of quite a few basic types:

- Capacitor-start induction motors
- Two-value capacitor induction motors
- Permanent-split capacitor induction motors
- Tapped-winding capacitor induction motors
- Split-phase capacitor induction motors
- Capacitor three phase induction motors (for single-phase supply)

23.3.1 Capacitor-start induction motors

The capacitor-start IM (Figure 23.2a) has a capacitor and a start switch in series with the auxiliary winding.

The starting capacitor produces an almost 90° phase advance of its current I_a with respect to I_m in the main winding (Figure 23.2b).

This way a large travelling field is produced at the start. Consequently, the starting torque is large. After the motor starts, the auxiliary winding circuit is opened by the starting switch, leaving on only the main winding. The large value of the starting capacitor is not adequate for running conditions. During running conditions, the power factor and efficiency are rather low.

Capacitor start motors are built for single or dual voltage (115 V and 230 V). For dual voltage, the main winding is built in two sections connected in series for 230 V and in parallel for 115 V.

23.3.2 The two-value capacitor induction motor

The two-value capacitor induction motor makes use of two capacitors in parallel—one for starting, C_s , and one for running C_n ($C_s \gg C_n$).

The starting capacitor is turned off by a starting switch while the running capacitor remains in series with the auxiliary winding (Figure 23.3).

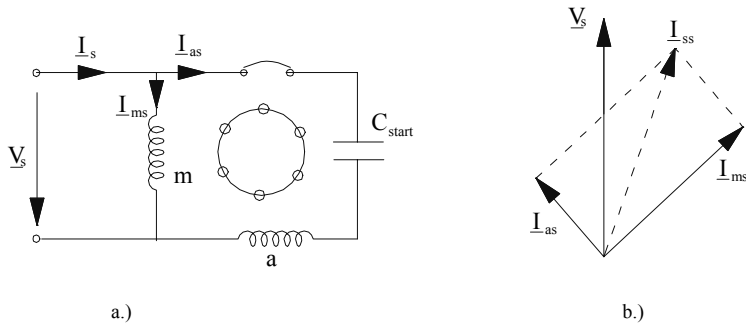


Figure 23.2 The capacitor-start induction motor
a.) schematics b.) phasor diagram at zero speed

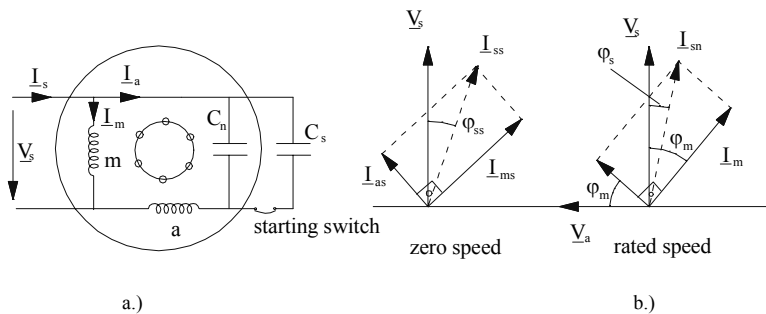


Figure 23.3 The two-value capacitor induction motor
a.) the schematics b.) phasor diagrams at zero and rated speed

The two capacitors are sized to symmetrize the two windings at standstill (C_s) and at rated speed (C_n).

High starting torque is associated with good running performance around rated speed (torque). Both power factor and efficiency are high.

The running capacitor is known to produce effects such as [1]

- 5 to 30% increase of breakdown torque
- 5-10% improvement in efficiency
- Power factors values above 90%
- Noise reduction at load
- 5-20% increase in the locked-rotor torque

Sometimes, the auxiliary winding is made of two sections (a_1 and a_2). One section works at start with the starting capacitor while both sections work for running conditions to allow a higher voltage (smaller) running capacitor (Figure 23.4).

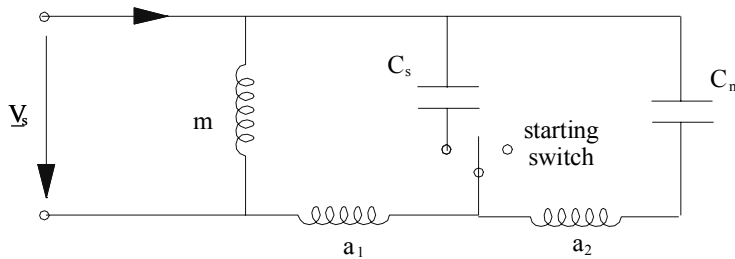


Figure 23.4 Two-value capacitor induction motor with two-section auxiliary winding

23.3.3 Permanent-split capacitor induction motors

The permanent-split capacitor IMs have only one capacitor in the auxiliary winding which remains in action all the time.

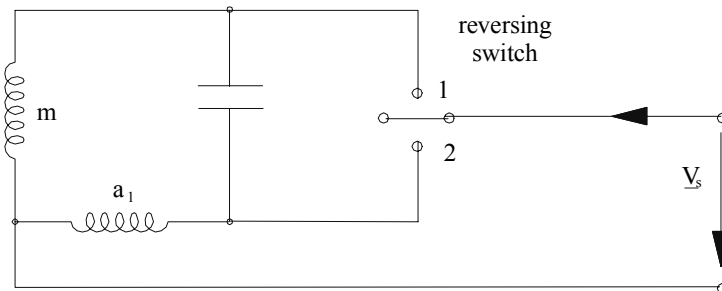


Figure 23.5 The reversing permanent-split capacitor motor

As a compromise between starting and running performance has to be performed, this motor has a rather low starting torque but good power factor and efficiency under load. This motor is not to be used with belt-transmissions due

to its low starting torque. It is however suitable for reversing, intermittent-duty service, and powers from 1 W to 200 W. For speed reversal the two windings are identical and the capacitor is “moved” from the auxiliary to the main winding circuit (Figure 23.5)

23.3.4 Tapped-winding capacitor induction motors

Tapped-winding capacitor induction motors are used when two or more speeds are required.

For two speeds the T and L connections are highly representative (Figure 23.6 a,b). The main winding contains two sections m_1 and m_2 .

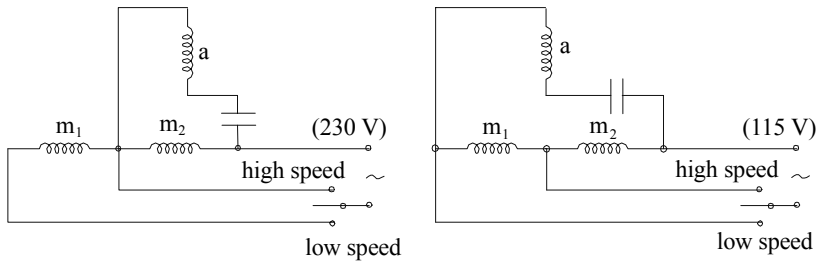


Figure 23.6 T(a) and L(b) connections of tapped winding capacitor induction motors

The T connection is more suitable for 230 V while the L connection is more adequate for 115V power grids since this way the capacitor voltage is higher and thus its cost is lower.

The difference between high speed and low speed is not very important unless the power of the motor is very small, because only the voltage is reduced. The locked rotor torque is necessarily low (less than the load torque at the low speed). Consequently, they are not to be used with belted drives. Unstable low speed operation may occur as the breakdown torque decreases with voltage squared.

A rather general tapped-winding capacitor induction motor is shown in Figure 23.7 [2]

In this configuration, each coil of the single layer auxiliary winding has a few taps corresponding to the number of speeds required.

23.3.5 Split-phase capacitor induction motors

Split-phase capacitor induction motors start as split-phase motors and then commute to permanent-capacitor motors.

This way both higher starting torque and good running performance is obtained. The auxiliary winding may also contain two sections to provide higher capacitance voltage. (Figure 23.8)

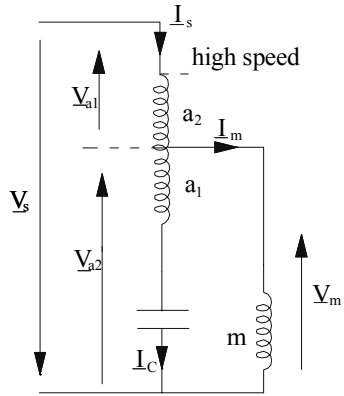


Figure 23.7 General tapped-winding . Capacitor induction motor

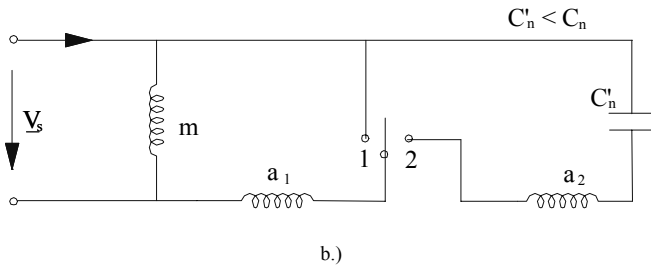
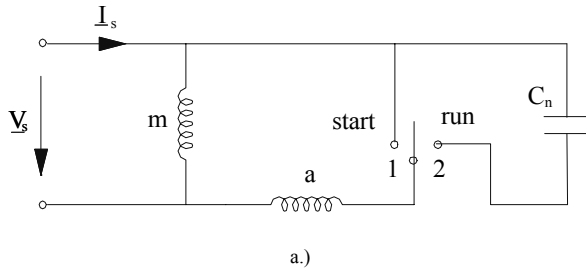


Figure 23.8 The split-phase capacitor induction motor
a.) with single section auxiliary winding; b.) with two section auxiliary winding

23.3.6 Capacitor three-phase induction motors

The three phase winding induction motors may be connected and provided with a capacitor to comply with single-phase power supplies. The typical connections also called Steinmetz star and delta connections-are shown in Figure 23.9. As seen in Figure 23.9 c), with an adequate capacitance C_n , the voltage of the three phases A,B,C may be made symmetric for a certain value of slip ($S = 1$ or $S' = S_n$ or a value in between).

The star connection prevents the occurrence of third m.m.f. space harmonic and thus its torque-speed curve does not show the large dip at 33% synchronous speed, accentuated by saturation and typical for low power capacitor single phase induction motors with two windings. Not so for delta connection (Figure 23.9 b) which allows for the third m.m.f. space harmonic. However, the delta connection provides $\sqrt{3}$ times larger voltage per phase and thus, basically, 3 times more torque in comparison with the star connection, for same single phase supply voltage.

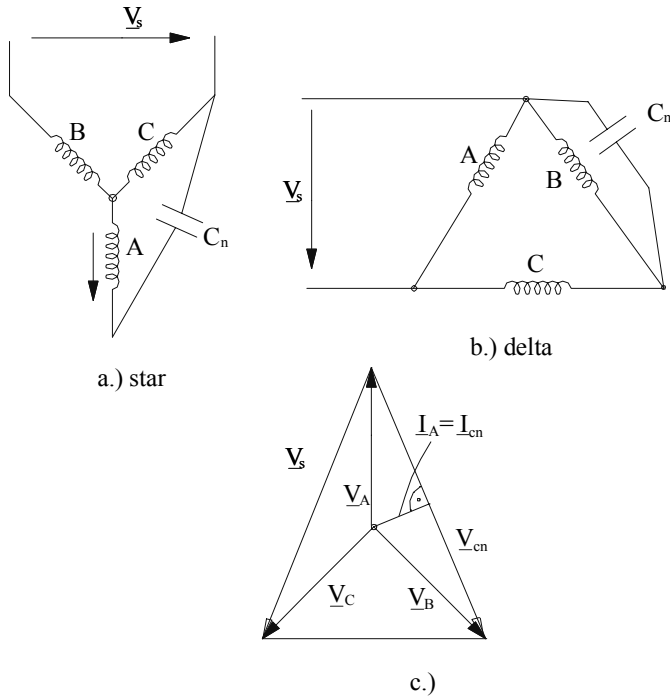


Figure 23.9 Three phase winding IM with capacitors for single phase supply
a.) star; b.) delta; c.) phasor diagram (star)

The three phase windings also allow for pole changing to reduce the speed in a 2:1 or other ratios (see Chapter 4)-Figure 23.10.

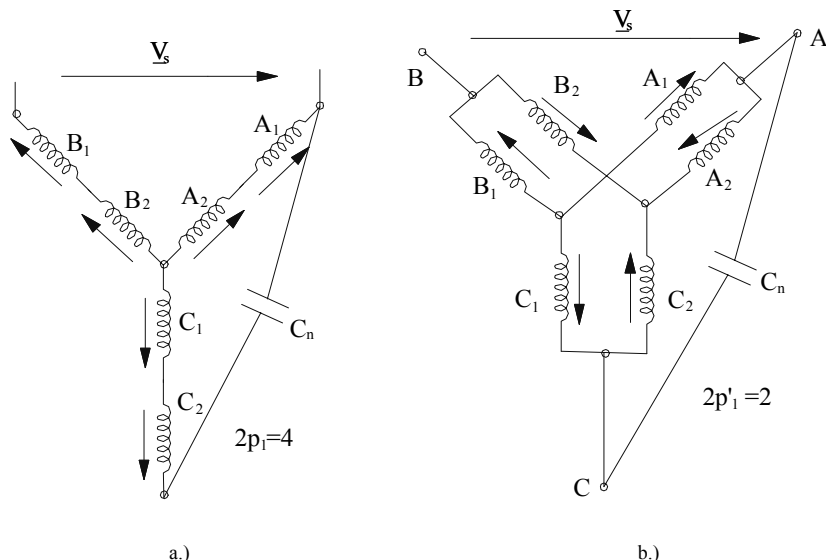


Figure 23.10 Pole-changing single phase induction motor with three phase stator winding
a.) $2p_1 = 4$ poles; b.) $2p_1' = 2$ poles

23.3.7 Shaded-pole induction motors

The shaded-pole induction motor has a single, concentrated coil, stator winding, with $2p_1 = 2, 4, 6$ poles, connected to a single phase a.c. power grid. To provide for selfstarting a displaced shortcircuited winding is located on a part of main winding poles (Figure 23.11).

The main winding current flux induces a voltage in the shaded-pole coil which in turn produces a current which affects the total flux in the shaded area. So the flux in the un-shaded area Φ_{mm} is both space-wise and time-wise shifted ahead with respect to the flux in the shaded area (Φ_a).

These two fluxes will produce a travelling field component that will move the rotor from the unshaded to the shaded area. Some further improvement may be obtained by increasing the airgap at the entry end of stator poles (Figure 23.11). This beneficial effect is due to the third space harmonic field reduction.

As the space distribution of airgap field is far from a sinusoid, space-harmonic parasitic asynchronous torques occur. The third harmonic is in general the largest and produces a notable dip in the torque-speed curve around 33% of synchronous speed $n_1 = f_1/p_1$.

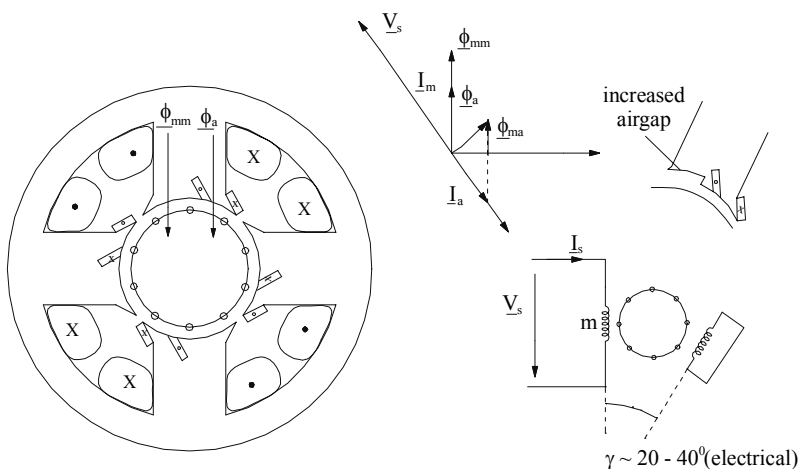


Figure 23.11 The shaded-pole induction motor

The shaded-pole induction motor is low cost but also has low starting torque, torque density, low efficiency (less than 30%), and power factor. Due to these demerits is used only at low powers between 1/25 to 1/5 kW in round frames (Figure 23.11), and less and less.

For even lower power levels (tens of mW), an unsymmetrical (C-shape) stator configuration is used (Figure 23.12).

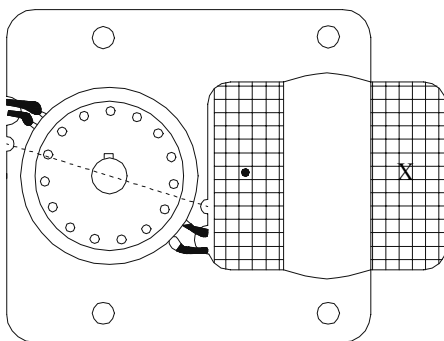


Figure 23.12 Two-pole C-shape shaded-pole induction motor

23.4 THE NATURE OF STATOR-PRODUCED AIRGAP FIELD

The airgap field distribution in single phase IMs, for zero rotor currents, depends on the distribution of the two stator windings in slots and the amplitude ratio I_m/I_a and phase shift γ of phase currents.

When the current waveforms are not sinusoidal in time, due to magnetic saturation or due to power grid voltage time harmonics, the airgap field

distribution gets even more complicated. For the time being, let us neglect the stator current time harmonics and consider zero rotor current.

Chapter 4 presents in some detail typical distributed windings for capacitor single phase induction motors.

Let us consider here the first and the third space harmonic of stator winding m.m.fs.

$$F_m(\theta_{es}, t) = F_{1m} \cos \omega_1 t \left[\cos \theta_{es} + \frac{F_{3m}}{F_{1m}} \cos(3\theta_{es}) \right] \quad (23.1)$$

$$F_a(\theta_{es}, t) = -F_{1a} \cos(\omega_1 t + \gamma_i) \left[\sin \theta_{es} + \frac{F_{3a}}{F_{1a}} \sin(3\theta_{es}) \right] \quad (23.2)$$

θ_{es} -stator position electrical angle.

For windings with dedicated slots, and coils with same number of turns, expressions of F_{1m} , F_{3m} , F_{1a} , F_{3a} (see Chapter 4) are

$$F_{1mv} = \frac{2N_m K_{w_{mv}} I_m \sqrt{2}}{\pi v p_1}; v = 1, 3 \quad (23.3)$$

$$F_{1av} = \frac{2N_a K_{w_{av}} I_a \sqrt{2}}{\pi v p_1}; v = 1, 3$$

The winding factors $K_{w_{mv}}$, $K_{w_{av}}$ depend on the relative number of slots for the main and auxiliary windings (S_m/S_a) and on the total number of stator slots S_1 (see Chapter 4).

For example in a 4 pole motor with $S_1 = 24$ stator slots, 16 slots for the main winding and 8 slots for the auxiliary winding, both of single layer type, the coils are full pitch. Also $q_m = 4$ slots/pole/phase for the main winding and $q_a = 2$ slots/pole/phase for the auxiliary winding. Consequently,

$$K_{w_{mv}} = \frac{\sin q_m \frac{\alpha_{cv}}{2}}{q_m \sin \frac{\alpha_{cv}}{2}} = \frac{\sin 4 \cdot \frac{\pi}{24} v}{4 \sin \frac{\pi}{24} v} \quad (23.4)$$

$$K_{w_{av}} = \frac{\sin q_a \frac{\alpha_{cv}}{2}}{q_a \sin \frac{\alpha_{cv}}{2}} = \frac{\sin 2 \cdot \frac{\pi}{24} v}{2 \sin \frac{\pi}{24} v} \quad (23.5)$$

$$F_{3m}/F_{1m} = 0.226; F_{3a}/F_{1a} = 0.3108$$

Let us first neglect the influence of slot openings while the magnetic saturation is considered to produce only an “increased apparent airgap.”

Consequently, the airgap magnetic field $B_{g0}(\theta_{er}, t)$ is:

$$B_{g0}(\theta_{es}, t) = \frac{\mu_0}{gK_c(1 + K_s)} [F_m(\theta_{es}, t) + F_a(\theta_{es}, t)] \quad (23.6)$$

K_s -saturation coefficients, K_c -Carter coefficient.

So, even in the absence of slot-opening and saturation influence on airgap magnetic permeance, the airgap field contains a third space harmonic. The presence of slot openings produces harmonics in the airgap permeance (see Chapter 10, paragraphs 10.3). As in single phase IM both stator and rotor slots are semiclosed, these harmonics may be neglected in a first order analysis.

On the other hand, magnetic saturation of slot neck leakage path produces a second order harmonic in the airgap permeance (Chapter 10; paragraph 10.4). This, in turn, is small unless the currents are large (starting conditions).

The main flux path magnetic saturation tends to create a second order airgap magnetic conductance harmonic (Chapter 10, paragraph 10.5),

$$\lambda_{gs}(\theta_{es}) = \frac{1}{gK_c} \left[\frac{1}{1 + K_{s1}} + \frac{1}{1 + K_{s2}} \sin \left(2 \frac{\theta_{es}}{p_1} - \omega_{lt} - \gamma_s \right) \right] \quad (23.7)$$

K_{s1} , K_{s2} are saturation coefficients that result from the decomposition of main flux distribution in the airgap.

We should mention here that rotor eccentricity also “induces” airgap magnetic conductance harmonics (Chapter 10, paragraph 10.7).

For example, the static eccentricity $\varepsilon = e/g$ yields

$$\lambda_{ge}(\theta_{es}) \approx \frac{1}{g} \left(C_0 + C_1 \cos \frac{\theta_{es}}{p_1} \right) \quad (23.8)$$

$$C_0 = \frac{1}{\sqrt{1 - \xi^2}}; C_1 = 2(C_0 - 1)/\xi$$

Now the airgap flux density $B_{g0}(\theta_{es}, t)$, for zero rotor currents, becomes

$$B_{g0}(\theta_{es}, t) = \mu_0 \lambda_{gs}(\theta_{es}) [F_m(\theta_{es}, t) + F_a(\theta_{es}, t)] \quad (23.9)$$

As seen from (23.9) and (23.1)-(23.2) the main flux path saturation introduces an additional third space harmonic airgap field, besides that produced by the m.m.f. third space harmonic. The two third harmonic field components are phase shifted with an angle γ_3 dependent on the relative ratio of the main and auxiliary windings m.m.f. amplitudes and on their time phase lag.

The third space harmonic of airgap field is expected to produce a third order notable asynchronous parasitic torque whose synchronism occurs around 1/3 of ideal no load speed $n_1 = f_1/p_1$. A rather complete solution of the airgap field distribution, for zero rotor currents (the rotor does not have the cage in slots) may be obtained by FEM.

In face to such complex field distributions in time and space, simplified approaches have traditionally become widely accepted.

Let us suppose that magnetic saturation and m.m.f. produced third harmonics are neglected. That is, we return to (23.6).

23.5 THE FUNDAMENTAL M.M.F. AND ITS ELLIPTIC WAVE

Neglecting the third harmonic in (23.1)-(23.2), the main and auxiliary winding m.m.fs are:

$$F_m(\theta_{es}, t) = F_{lm} \cos \omega_{lt} \cdot \cos \theta_{es} \quad (23.10)$$

$$F_a(\theta_{es}, t) = -F_{la} \cos(\omega_{lt} + \gamma_i) \sin \theta_{es} \quad (23.11)$$

F_{la}/F_{lm} is, from (23.3)

$$F_{la} / F_{lm} = \frac{N_m K_{wml}}{N_a K_{wal}} \cdot \frac{I_m}{I_a} = \frac{1}{a} \frac{I_m}{I_a} \quad (23.12)$$

Given the sinusoidal spatial distribution of the two m.m.fs, their resultant space vector $F(t)$ may be written as

$$F(t) = F_{lm} \sin \omega_{lt} + jF_{la} \sin(\omega_{lt} + \gamma_i) \quad (23.13)$$

So the amplitude $F(t)$ and its angle $\gamma(t)$ are

$$\begin{aligned} F(t) &= \sqrt{F_{lm}^2 \cos^2 \omega_{lt} + F_{la}^2 \sin^2(\omega_{lt} + \gamma_i)} \\ \gamma(t) &= \tan^{-1} \left[-a \frac{I_a}{I_m} \frac{\cos(\omega_{lt} + \gamma_i)}{\cos \omega_{lt}} \right] \end{aligned} \quad (23.14)$$

A graphical representation of (23.13) in a plane with the real axis along the main winding axis and the imaginary one along the auxiliary winding axis, is shown in [Figure 23.13](#). The elliptic hodograph is evident.

Both the amplitude $F(t)$ and time derivative ($d\gamma/dt$) (speed) vary in time from a maximum to a minimum value.

Only for symmetrical condition, when the two m.m.f.s have equal amplitudes and are time-shifted by $\gamma_i = 90^\circ$, the hodograph of the resultant m.m.f. is a circle. That is, a pure **travelling** wave ($\gamma = \omega_{lt}$) is obtained. It may be demonstrated that the m.m.f. wave speed $d\gamma/dt$ is positive for $\gamma_i > 0$ and negative for $\gamma_i < 0$.

In other words, the motor speed may be reversed by changing the sign of the time phase shift angle between the currents in the two windings. One way to do it is to switch the capacitor from auxiliary to main winding ([Figure 23.5](#)).

The airgap field is proportional to the resultant m.m.f. (23.6), so the elliptic hodograph, of m.m.f. stands valid for this case also.

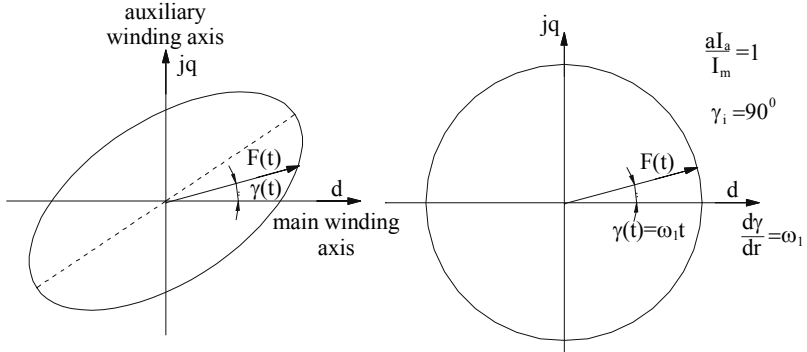


Figure 23.13 The hodograph of fundamental m.m.f. in single phase IMs
a.) in general b.) for symmetrical conditions

When heavy magnetic saturation occurs for sinusoidal voltage supply, zero rotor currents, the stator currents become nonsinusoidal, while the e.m.f. phase flux linkages, also depart from sinusoids.

The presence of rotor currents further complicates the picture. It is thus fair to say that the computation of field distribution, currents, power, torque for the single phase IMs is more complicated than for three-phase IMs. A saturation model will be introduced in Chapter 24.

23.6 FORWARD-BACKWARD M.M.F. WAVES

The resultant stator m.m.f. $F(\theta_{es}, t)$ -(23.10-23.11)-may be decomposed in two waves

$$\begin{aligned} F(\theta_{es}, t) &= F_{lm} \cos \omega_1 t \cos \theta_{es} + F_{la} \cos(\omega_1 t + \gamma_i) \sin \theta_{es} = \\ &= \frac{1}{2} F_m C_f \sin(\theta_{es} - \omega_1 t - \beta_f) + \frac{1}{2} F_m C_b \sin(\theta_{es} + \omega_1 t - \beta_b) \end{aligned} \quad (23.15)$$

$$\begin{aligned} C_f &= \sqrt{\left(1 + \frac{F_{la}}{F_{lm}} \sin \gamma_i\right)^2 + \left(\frac{F_{la}}{F_{lm}}\right)^2 \cos^2 \gamma_i} \\ \sin \beta_f &= \frac{(1 + F_{la} \sin \gamma_i / F_{lm})}{C_f} \end{aligned} \quad (23.16)$$

$$\begin{aligned} C_b &= \sqrt{\left(1 - \frac{F_{la}}{F_{lm}} \sin \gamma_i\right)^2 + \left(\frac{F_{la}}{F_{lm}} \cos \gamma_i\right)^2} \\ \sin \beta_b &= \frac{(1 - F_{la} \sin \gamma_i / F_{lm})}{C_b} \end{aligned} \quad (23.17)$$

Again, for $F_{1a}/F_{1m} = 1$ and $\gamma_i = 90^\circ$, $C_b = 0$ and thus the backward (b) travelling wave becomes zero (the case of circular hodograph in [Figure 23.13 b](#)).

Also, the forward (f) travelling amplitude, is in this case, equal to $F_{1m} = F_{1a}$ (it is $3/2 F_{1m}$ for a three phase symmetrical winding).

For $F_{1a} = 0$, the f.b. waves have the same magnitude: $\frac{1}{2} F_{1m}$.

The m.m.f. decomposition in forward and backward waves may be done even with saturation while the superposition of the forward and backward field waves is correct only in the absence of magnetic saturation.

The fb decomposition is very practical as it leads to simple equivalent circuits with the slip $S_f = S$ and, respectively, $S_b = 2-S$. This is how the travelling (fb)-revolving field theory of single phase IMs evolved.

$$S_f = \frac{\omega_1 - \omega_r}{\omega_r} = S; S_b = \frac{(-\omega_1) - \omega_r}{(-\omega_1)} = 2 - S \quad (23.18)$$

For steady state, the fb model is similar to the symmetrical components model. On the other hand, for the general case of transients or nonsinusoidal voltage supply, the dq cross-field model has become standard.

23.7 THE SYMMETRICAL COMPONENTS GENERAL MODEL

For steady state and sinusoidal currents, time phasors may be used. The unsymmetrical m.m.f., currents and voltages corresponding to the two windings of single phase IMs may be decomposed in two symmetrical systems ([Figure 23.14](#)) which are in fact the forward and backward components introduced in the previous paragraph.

It goes without saying that the two windings are 90° electrical degrees phase shifted spatially.

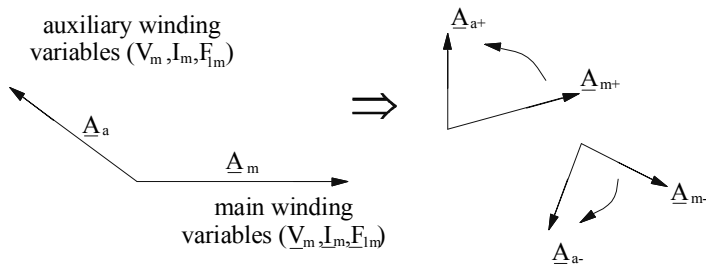


Figure 23.14 Symmetrical (fb or + -) components of a two phase winding motor

From [Figure 23.14](#)

$$\underline{A}_{a+} = j\underline{A}_{m+}; \underline{A}_{a-} = -j\underline{A}_{m-} \quad (23.19)$$

The superposition principle yields

$$\begin{aligned}\underline{A}_m &= \underline{A}_{m+} + \underline{A}_{m-} \\ \underline{A}_f &= \underline{A}_{a+} + \underline{A}_{a-}\end{aligned}\quad (23.20)$$

From (23.19)-(23.20),

$$\underline{A}_{m+} = \frac{1}{2}(\underline{A}_m - j\underline{A}_a) \quad (23.21)$$

$$\underline{A}_{m-} = \frac{1}{2}(\underline{A}_m + j\underline{A}_a) = \underline{A}_{m+}^* \quad (23.22)$$

Denoting by $\underline{V}_{m+}$, $\underline{V}_{m-}$, $\underline{I}_{m+}$, $\underline{I}_{m-}$ and respectively $\underline{V}_{a+}$, $\underline{V}_{a-}$, $\underline{I}_{a+}$, $\underline{I}_{a-}$, - the voltage and current components of the two winding circuits, we may write the equations of the two fictitious 2-phase symmetric machines (whose travelling fields are opposite (f,b)) as

$$\begin{aligned}\underline{V}_{m+} &= \underline{Z}_{m+} \underline{I}_{m+}; \underline{V}_{m-} = \underline{Z}_{m-} \underline{I}_{m-} \\ \underline{V}_{a+} &= \underline{Z}_{a+} \underline{I}_{a+}; \underline{V}_{a-} = \underline{Z}_{a-} \underline{I}_{a-} \\ \underline{V}_m &= \underline{V}_{m+} + \underline{V}_{m-}; \underline{V}_a = \underline{V}_{a+} + \underline{V}_{a-}\end{aligned}\quad (23.23)$$

The $\underline{Z}_{m+}$, $\underline{Z}_{m-}$ and respectively $\underline{Z}_{a+}$, $\underline{Z}_{a-}$ represent the resultant forward and backward standard impedances of the fictitious induction machine, per phase, with the rotor cage circuit reduced to the m, and respectively, a stator winding (Figure 23.15).

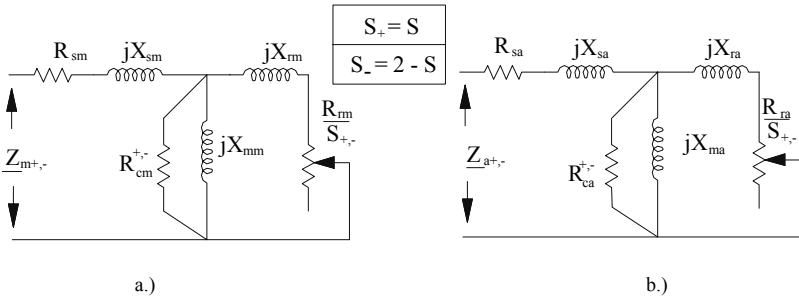


Figure 23.15. Equivalent symmetrical (+,-) impedances

a.) reduced to the main stator winding m; b.) reduced to the auxiliary stator winding a

We still need to solve Equations (23.33), with given slip S value and all parameters in $\underline{Z}_{m+,-}$ and $\underline{Z}_{a+,-}$. A relationship between the $\underline{V}_m$, $\underline{V}_a$ and the source voltage $\underline{V}_s$ is needed.

When an impedance ($\underline{Z}_a$) is connected in series with the auxiliary winding and then both windings are connected to the power grid given voltage $\underline{V}_s$ we obtain a simple relationship:

$$\underline{V}_m = \underline{V}_s; \underline{V}_a + (\underline{I}_{a+} + \underline{I}_{a-})\underline{Z}_a = \underline{V}_s \quad (23.24)$$

It is generally more comfortable to reduce all windings on rotor and on stator to the main winding.

The reduction ratio of auxiliary to main winding is the coefficient a in Equation (23.12)

$$a = \frac{N_a}{N_m} \frac{K_{w1a}}{K_{w1m}} \quad (23.25)$$

The rotor circuit is symmetric when $a^2 X_{rm} = X_{ra}$, $a^2 R_{rm} = R_{ra}$.

Consequently, when

$$\begin{aligned} \Delta X_{sa} &= X_{sa} - a^2 X_{sm} = 0 \\ \Delta R_{sa} &= R_{sa} - a^2 R_{sm} = 0 \end{aligned} \quad (23.26)$$

the two equivalent circuits on [Figure 23.15](#), both reduced to the main winding, become identical.

This situation occurs in general when both windings occupy the same number of uniform slots and the design current density for both windings is the same. When the auxiliary winding occupies 1/3 of stator periphery, its design current density may be higher to fulfill (23.26).

The voltage and current reduction formulas from the auxiliary winding to main winding are

$$\underline{V}'_a = \underline{V}_a / a; \underline{I}'_a = a \underline{I}_a \quad (23.27)$$

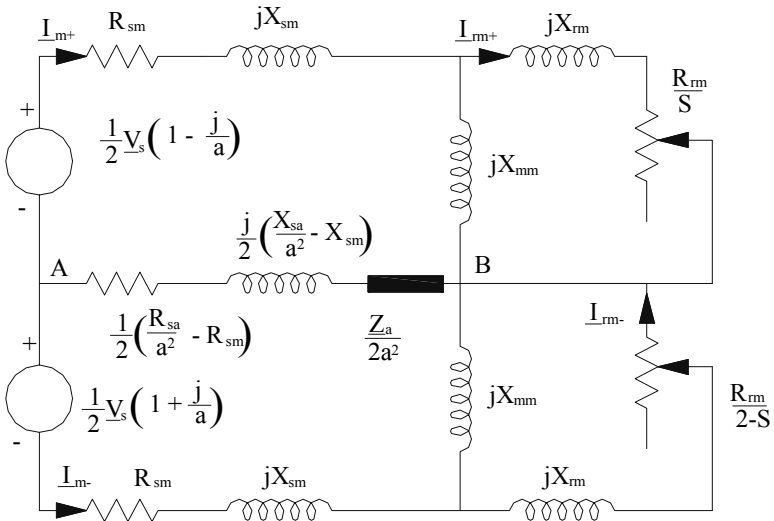


Figure 23.16 General equivalent circuit of split-phase and capacitor induction motor

In such conditions, when $\Delta X_{sa} \neq 0$ and $\Delta R_{sa} \neq 0$, their values are introduced in series with the capacitor impedance $\underline{Z}_a$ after reduction to the main winding and division by 2. The division by 2 is there to conserve powers (Figure 23.16).

Note again that $\underline{Z}_a$ contains the capacitor. In split-phase IMs, $\underline{Z}_a = 0$. When the auxiliary winding is open: $\underline{Z}_a = \infty$. Also note that the $+$ - voltages are now defined at the grid terminals ($V_a = V_s/a$) rather than at the winding terminals.

This is how the A to B section impedance on Figure 23.16 takes its place inside the equivalent circuit. The equivalent circuit in Figure 23.16 is fairly general. It has only two unknowns I_{m+} and I_{m-} and may be solved rather easily for given slip, machine parameters and supply voltage $\underline{V}_s$.

Tapped winding (multiple-section winding or three phase winding) capacitor IMs may be represented directly by the equivalent circuit in Figure 23.16. Core loss resistances may be added in Figure 23.16 in parallel with the magnetization reactance X_{mm} or with the leakage reactances (for additional losses) but the problem is how to calculate (measure) them.

Main flux path saturation may be included by making the magnetization reactance X_{mm} a variable. The first temptation is to consider that only the forward (direct component) X_{mm} (the upper part of Figure 23.16) saturates, but, as shown earlier, the elliptic m.m.f. makes the matters far more involved.

This is why the general equivalent circuit in Figure 23.16 is suitable when the saturation level is constant or it is neglected.

To complete the picture we add here the average torque expression

$$T_e = T_{e+} + T_{e-} = \frac{2p_1}{\omega_1} \left[I_{m+}^2 \cdot \frac{R_{rm}}{S} - \frac{I_{m-}^2 \cdot R_{rm}}{2 - S} \right] \quad (23.28)$$

For the case when the auxiliary winding is open ($\underline{Z}_a = \infty$), $I_{m+} = I_{m-}$. In such a case for $S = 1$ (start), the total torque T_e is zero, as expected.

It may be argued that as the rotor currents experience two frequencies: $f_{2+} = Sf_1$ and $f_{2-} = (2 - S)f_1 > f_1$, the rotor cage parameters differ for the two components due to skin effects.

To account for skin effect, two fictitious cages in parallel may be considered in Figure 23.18. Their parameters are chosen to fit the rotor impedance variation from $f_2 = 0$ to $f_2 = 2f$ according to a certain optimization criterion.

23.8 THE d-q MODEL

As the stator is provided with two orthogonal windings and the rotor is fully symmetric, the single phase IM is suitable for the direct application of dq model in stator coordinates (Figure 23.17).

The rotor will be reduced here to the main winding, while the auxiliary winding is not reduced to the main winding.

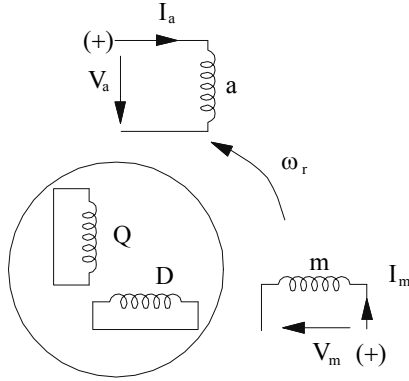


Figure 23.17 The d-q model of single phase IM

The dq model equations in stator coordinates are straightforward

$$\begin{aligned}
 I_m R_{sm} - V_m &= -\frac{d\psi_m}{dt} \\
 I_a R_{sa} - V_a &= -\frac{d\psi_a}{dt} \\
 i_D R_r &= -\frac{d\psi_D}{dt} - \omega_r \psi_a \\
 I_Q R_r &= -\frac{d\psi_a}{dt} + \omega_r \psi_D
 \end{aligned} \tag{23.29}$$

The flux current relationships are

$$\begin{aligned}
 \psi_D &= L_{rm} I_D + L_{mm} (I_m + I_D) \\
 \psi_Q &= L_{rm} I_Q + L_{am} (a I_q + I_Q) / a^2 \\
 \psi_m &= L_{sm} I_m + L_{mm} (I_m + I_D) \\
 \psi_a &= L_{sa} I_a + L_{am} \left(I_a + \frac{I_Q}{a} \right)
 \end{aligned} \tag{23.30}$$

For nonlinear saturation conditions, it is better to define a the reduction ratio as

$$a = \sqrt{\frac{L_{am}}{L_{mm}}} \tag{23.31}$$

This aspect will be treated in a separate chapter on testing (Chapter 28). The magnetization inductances L_{mm} and L_{am} along the two axes (windings) are dependent on the resultant magnetization current I_μ

$$I_{\mu}(t) = \sqrt{(I_m + I_D)^2 + (aI_a + I_Q)^2} \quad (23.32)$$

The torque and motion equations are straightforward.

$$T_e = -p_1(\psi_D I_Q - \psi_Q I_D) = p_1 L_{mm} \left(\sqrt{\frac{L_{am}}{L_{mm}}} i_a I_D - I_m I_Q \right) \quad (23.33)$$

$$\frac{J}{p_1} \frac{d\omega_r}{dt} = T_e - T_{load}(\omega_r, \theta_{er}, t) \quad (23.34)$$

Still missing are the relationships between the stator voltages V_m , V_a and the source voltage V_s . In a capacitor IM

$$\begin{aligned} V_m(t) &= V_s(t) \\ V_a(t) &= V_s(t) - V_c(t) \end{aligned} \quad (23.35)$$

$$C \frac{dV_c(t)}{dt} = I_a(t) \quad (23.36)$$

V_c is the capacitor voltage.

A six order nonlinear model has been obtained. The variables are: I_m , I_a , I_D , I_Q , ω_r , V_c , while the source voltage $V_s(t)$ and the load torque $T_{load}(\omega_r, \theta_{er}, t)$ constitute the inputs.

With known $L_{am}(I_{\mu})$ and $L_{mm}(I_{\mu})$ functions even the saturation presence may be handled elegantly, both during transients and steady state.

23.9 THE d-q MODEL OF STAR STEINMETZ CONNECTION

The three phase winding IM, in the capacitor motor connection for single phase supply, may be reduced simply to the d-q (m,a) model as

$$V_m + jV_a = \sqrt{\frac{2}{3}} \left(V_a(t) + V_b(t) e^{j\frac{2\pi}{3}} + V_c(t) e^{-j\frac{2\pi}{3}} \right) e^{-j\theta_0} \quad (23.37)$$

The same transformation is valid for currents while the phase resistances and leakage inductances stand unchanged.

The voltage relationships depend on the connection of phases. For the Steinmetz star connection (Figure 23.18),

$$\begin{aligned}
V_B(t) - V_C(t) &= V_S(t) \\
V_a(t) + V_b(t) + V_c(t) &= 0 \\
V_B(t) - V_A(t) &= V_{\text{cap}}(t) \\
I_A(t) &= I_{\text{cap}}(t) \\
C \frac{dV_{\text{cap}}}{dt} &= I_{\text{cap}}(t)
\end{aligned} \tag{23.38}$$

With $\theta_0 = -\pi/2$, d axis falls along the axis of the main winding

$$\begin{aligned}
V_m(t) &= \frac{1}{\sqrt{2}}(V_B - V_C) = \frac{V_S(t)}{\sqrt{2}} \\
V_a(t) &= +\sqrt{\frac{3}{2}}V_A(t)
\end{aligned} \tag{23.39}$$

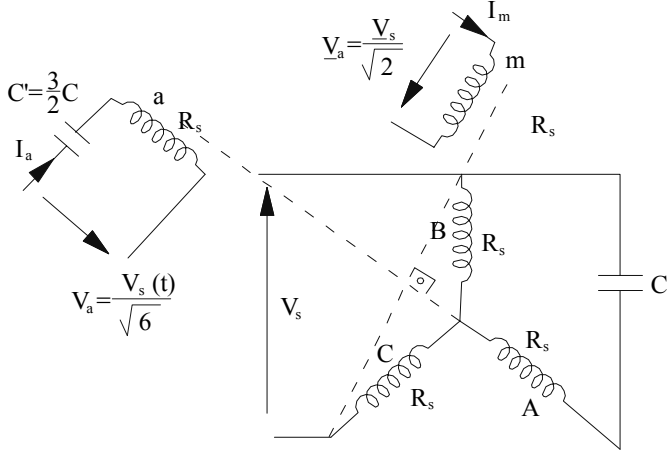


Figure 23.18 Three (two) phase equivalence

From (23.38) with (23.39)

$$\sqrt{\frac{2}{3}}V_{\text{cap}} + V_a = \frac{V_S(t)}{\sqrt{6}} \tag{23.40}$$

Notice that

$$I_a = \sqrt{\frac{3}{2}}I_{\text{cap}} \tag{23.41}$$

For the d-q model equation (23.40) becomes

$$V'_{\text{cap}} + V_a = \frac{V_s(t)}{\sqrt{6}}$$

$$\frac{dV'_{\text{cap}}}{dt} = \frac{1}{C} I_a$$
(23.42)

Also,

$$V'_{\text{cap}} = \frac{1}{C} \cdot \int I_a dt = \sqrt{\frac{2}{3}} \frac{1}{C} \int I_a dt = \frac{2}{3C} \int I_a dt = \frac{1}{\left(\frac{3}{2}C\right)} \int I_a dt$$
(23.43)

So,

$$C' = \frac{3}{2}C$$
(23.44)

The equivalent capacitance circuit C' in the d-q (m.a)-2 phase –model is 1.5 times higher than the actual capacitance C . The source voltage to the main winding m(d) is $V_s / \sqrt{2}$ instead of V_s as it is for the usual model. For the auxiliary winding the “virtual source” voltage is $V_s / \sqrt{6}$ (Equation 23.40). Finally the resistance, leakage inductance and magnetization inductance are those of the three phase machine per phase. Consequently, for the m&a model stator windings are now identical.

The steady state performance may be approached by running transients from start (or given initial conditions) until conditions stabilize. As the state-space system of equations is solvable only via numerical methods, its solution takes time. With zero initial values for currents, capacitor voltage and given (final) slip (speed), this time is somewhat reduced.

Alternatively, it is possible to use $j\omega_1$ instead of d/dt in Equations (23.29) and to extract the real part of torque expression, after solving the algebraic system of equations for the currents and the capacitor voltage.

This approach may be more suitable than the travelling (revolving) field model when the magnetic saturation is to be accounted for, for the resultant (elliptical) magnetic field in the machine may be accounted for directly. However, it does not exhibit the intuitive attributes of the revolving field theory. In presence of strong computation facilities, this steady state approach to the dq model may revive for usage when magnetic saturation is heavy.

Finally, the reduction of the three-phase Steinmetz star connection to the m,a model performed above may also be used in the symmetrical components model (Figure 23.16).

23.10 SUMMARY

- Single phase supply induction motors have a cage rotor and two phase stator windings: the main winding and the auxiliary winding.
- The two windings are phaseshifted spatially by 90° electrical degrees.

- The auxiliary winding may have a larger resistance/reactance ratio or a series capacitance to push ahead its current phase angle with respect to that of the main winding.
- Alternatively, the auxiliary winding may be shortcircuited, but in this case its coils should occupy only a part of the main winding poles. This is the shaded-pole induction motor. It is low cost but also low performance. It is fabricated for powers below (1/5) kW and down to 10-15W.
- The capacitor induction motor may be built in capacitor-start, dual-capacitor permanent-capacitor, tapped winding capacitor, split-phase capacitor configurations. Each of these types is suitable for certain applications. Except for the capacitor-start IMs, all the other capacitor IMs have good efficiency and over 90% power factor but only capacitor-start, dual-capacitor and split-phase configurations exhibit large starting torque (up to 250% rated torque)
- Speed reversal is obtained by “moving” the capacitor from the auxiliary to the main winding circuit. In essence, this move reverses the predominant travelling field speed in the airgap.
- The three phase stator winding motor may be single-phase supply fed, adding a capacitor mainly in the so-called Steinmetz star connection. Notable derating occurs unless the three phase motor was not designed for delta connection.
- The nature of the airgap field in capacitor IMs may be easily investigated for zero rotor currents (open rotor cage). The two stator winding m.m.f.s and their summation contains a notable third space harmonic. So does the airgap flux. This effect is enhanced by the main flux path magnetic saturation which creates a kind of second space harmonic in the airgap magnetic conductance. Consequently, a notable third harmonic asynchronous parasitic torque occurs. A dip in the torque/speed curve around 33% of ideal no load speed shows up, especially for low powers motors (below 1/5 kW)
- Even in absence of magnetic saturation and slot opening effects, the stator m.m.f. fundamental produces an elliptic hodograph. This hodograph degenerates into a circle when the two phase m.m.f.s are perfectly symmetric (90° (electrical) winding angle shift, 90° current phase shift angle).
- The main (m) and auxiliary (a) m.m.f. fundamentals produce together elliptic wave which may be decomposed into a forward (+) and a backward (-) travelling wave.
- The backward wave is zero for perfect symmetrization of phases.
- For zero auxiliary m.m.f. (current), the forward and backward m.m.f. waves have equal magnitudes.
- The (+) (-) m.m.f. waves produce, by superposition, corresponding airgap field waves (saturation neglected). The slip is S for the forward wave and 2-S for the backward wave. This is a phenomenological basis for applying

symmetrical components to assess capacitor IM performance under steady state.

- The symmetrical components method leads to a fairly general equivalent circuit (Figure 23.16) which shows only two unknowns—the + and - components of main winding current, I_{m+} and I_{m-} , for given slip, supply voltage and machine parameters. The average torque expression is straightforward (23.18).
- When the auxiliary winding parameters R_{sa} , X_{sa} , reduced to the main winding R_{sa}/a^2 , X_{sa}/a^2 , differ from the main winding parameters R_{sm} , X_{sm} , the equivalent circuit (Figure 23.16) contains, in series with the capacitance impedance Z_a (reduced to the main winding Z_a/a^2) after division by 2, ΔR_{sa} and ΔX_{sa} . This fact brings a notable generality to the equivalent circuit in Figure 23.16.
- To treat the case of open auxiliary winding ($I_a = 0$), it is sufficient to put $Z_a = \infty$ in the equivalent circuit in Figure 23.16. In this case, the forward and backward main winding components are equal to each other and thus at zero speed ($S = 1$), the total torque (23.18) is zero, as expected.
- The dq model seems ideal for treating the transients of single phase induction motor. Cross-coupling saturation may be elegantly accounted for in the dq model. The steady state is either obtained after solving the steady state 6 order system via a numerical method at given speed for zero initial variable values. Alternatively, the solution may be found in complex numbers by putting $d/dt = j\omega_1$. The latter procedure though handy on the computer, in presence of saturation, does not show the intuitive attributes (circuits) of the travelling (revolving) field theory of symmetrical components.
- The star Steinmetz connection may be replaced by a 2 winding capacitor motor provided the capacitance is increased by 50%, the source voltage is reduced $\sqrt{2}$ times for the main winding and is $V_s / \sqrt{6}$ for the auxiliary one. The two winding parameters are equal to phase parameters of the three phase original machine. The star Steinmetz connection is lacking the third m.m.f. space harmonic. However, the third harmonic saturation produced asynchronous torque still holds.
- Next chapter deals in detail with steady state performance in detail.

23.11 REFERENCES

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